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S2_3 Colonising Neptune

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Abstract

The paper 'Getting comfortable on Neptune' [1] calculated the angular velocity that would be required to simulate an effective downwards acceleration of 9 ms^{-2} at the equator of Neptune, by partially counter acting the effects of gravity. This paper calculates the percentage of Neptune's surface that would fall within a region of 'habitable' downwards accelerations; estimated to be between 9 ms^{-2} and 10 ms^{-2}. This region was calculated to extend to a distance of 16,800 km from the centre of the planet along the axis of rotation, corresponding to 68% of the planets surface.

Introduction

This paper is a continuation of the 'Getting comfortable on Neptune' paper [1] which calculated the angular velocity required to make the centrifugal and Coriolis effects large enough to simulate an effective downwards acceleration of 9ms^{-2} at the equator of Neptune. However, these forces depend on the distance from the axis of rotation and will therefore, decrease with distance from the equatorial plane. As these forces decrease, the effective downwards acceleration would simultaneously increase. In this paper we aimed to determine the percentage of Neptune's surface that would have 'habitable' effective downwards accelerations; estimated to be between 9 ms^{-2} and 10 ms^{-2}.

Method

The effective downwards acceleration ($\boldsymbol{a_{eff}}$) at a point on the surface of a planet is given by [1]

$$\boldsymbol{a_{eff}} = \boldsymbol{g} - 3\boldsymbol{\omega}^2\boldsymbol{r}, \tag{1}$$

where $\boldsymbol{r}$ is perpendicular to the axis of rotation and equal to the distance from the axis to the surface, as shown in figure (1). $\boldsymbol{\omega}$ is the angular velocity of the planet, this value was calculated to be 1.70×10^{-4}rad s^{-1} in the preceding paper [1]. $\boldsymbol{g}$ is the acceleration due to gravity at the surface of Neptune. As the centrifugal and Coriolis effects (collectively described in the 2nd term on the right hand side of Equation (1)) are both proportional to $\boldsymbol{r}$, they will decrease with distance from the equatorial plane.

Distance $\boldsymbol{r}$ was substituted for $\sqrt{\boldsymbol{R}^2 - \boldsymbol{x}^2}$ where $\boldsymbol{x}$ is the distance along the rotational axis and $\boldsymbol{R}$ is the radius of Neptune (24,622km)[2]. Therefore, Equation (1) becomes;

$$\boldsymbol{a_e f f} = \boldsymbol{g} - 3\omega^2\sqrt{\boldsymbol{R}^2 - \boldsymbol{x}^2}, \tag{2}$$

which is represented by the green line in figure (2). However, the centrifugal and Coriolis effects are always perpendicular to the axis of rotation. Therefore, the component of the forces acting in a parallel and opposite direction to the gravitational force will be different. This is demon-

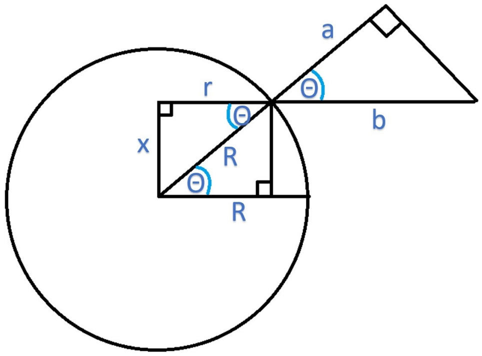

Figure 1: A diagram illustrating many of the terms used in this paper. '$\boldsymbol{b}$' represents the total centrifugal and Coriolis forces and '$\boldsymbol{a}$' represents the components of the forces that are parallel and opposite to the gravitational force at a given point on the surface.

strated in figure (1) where '$\boldsymbol{b}$' represents the total centrifugal and Coriolis forces and '$\boldsymbol{a}$' represents the component directly opposing gravity.

$$\boldsymbol{a} = \boldsymbol{b}cos(\theta) = \boldsymbol{b}\frac{\boldsymbol{r}}{\boldsymbol{R}} = \boldsymbol{b}\frac{\sqrt{\boldsymbol{R}^2 - \boldsymbol{x}^2}}{\boldsymbol{R}}. \tag{3}$$

The centrifugal and Coriolis term in Equation (2) represents the total force, which must be corrected by multiplying by $\frac{\sqrt{\boldsymbol{R}^2-\boldsymbol{x}^2}}{\boldsymbol{R}}$ to give;

$$\boldsymbol{a_e ff} = \boldsymbol{g} - 3\boldsymbol{\omega}^2\frac{\boldsymbol{R}^2 - \boldsymbol{x}^2}{R}. \tag{4}$$

This is shown by the blue line in figure (2). Rearranging Equation (4) allowed the vertical distance '$\boldsymbol{x}$' for a given effective acceleration to be found;

$$\boldsymbol{x} = \sqrt{\boldsymbol{R}^2 - \frac{\boldsymbol{g} - \boldsymbol{a_e ff}}{3\boldsymbol{\omega}^2}\boldsymbol{R}}. \tag{5}$$

This allowed the surface area of the planet that falls within a region of 9 ms^{-2} to 10 ms^{-2} to be calculated. The surface area (A_d) that lies outside this region is given by;

$$A_d = 2 \times \frac{1}{6}\boldsymbol{Rh} = \frac{1}{3}\boldsymbol{R}(\boldsymbol{R} - \boldsymbol{x}), \tag{6}$$

where the factor of two arises from having two hemispheres and $\boldsymbol{h}$ is the 'height' of the hemisphere. The total surface area is simply $A_s = 4\pi\boldsymbol{R}^2$, meaning that the fraction of the planet's surface area within the habitable region (A_H) is given by;

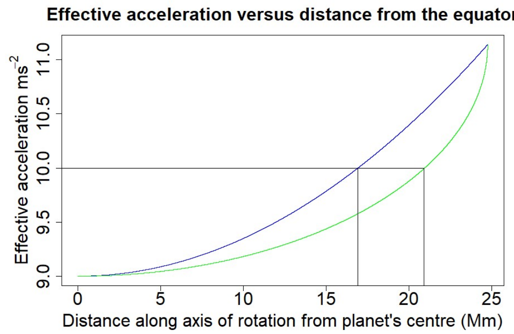

Figure 2: A graph showing how the effective downwards acceleration varies with distance from the equatorial plane. The green line represents the total effective acceleration and the blue line represents the component in the opposite direction to gravity. The black lines show the limit of the habitable region for both models.

$$A_H = 1 - \frac{4\pi\boldsymbol{R}^2}{\frac{1}{3}\boldsymbol{R}(\boldsymbol{R} - \sqrt{\boldsymbol{R}^2 - \frac{\boldsymbol{g}-\boldsymbol{a_e ff}}{3\boldsymbol{\omega}^2}\boldsymbol{R}})} = 0.68. \tag{7}$$

Conclusion

A region of habitable downwards accelerations was estimated to be between 9 ms^{-2} of 10 ms^{-2}. It was calculated that this region had limits along the axis of rotation ($\boldsymbol{x}$) of 16,800 km from the centre of the planet. This corresponded to 68% of Neptune's surface falling within this region. This area is approximately 10 times larger than the whole of the Earth's surface meaning colonisation through this process would be worthwhile if it proves to be viable.

References

[1] Tom Berry, Anthony Erojo, Ben Taylor. (2017). S2_3 Getting comfortable on Neptune

[2] https://www.space.com/18924-how-big-is-neptune.html Accessed 21/11/2017